Experimental validation of a cascaded wheel slip control strategy

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Design and experimental validation of a cascaded wheel slip control strategy

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Both maintaining the vehicle manoeuvrability during heavy braking and maximizing the brake force can be achieved by regulating the longitudinal wheel slip around an appropriate setpoint. In this paper, a new wheel slip controller is presented and validated experimentally. The control strategy is based on both wheel slip and wheel acceleration regulation through a cascaded approach; which was proven to be globally asymptotically stable in both the stable and unstable regions of the tyre. Simulations are available to assess the robustness against actuation delays and uncertainties in the tyre-road friction. Furthermore, tests on a tyre-in-the-loop facility show that the wheel slip do converge precisely to the assigned reference.

Topics / Traction and Brake Control; Tire Property ; Driver Assistance Systems.

1. INTRODUCTION

The purpose of anti-lock brake systems is twofold. On the one hand, their objective is to avoid wheel lock-up (in order to preserve the tyre ability of producing a lateral force, and thus vehicle maneuverability). On the other hand, they try to keep the wheel slip in a neighborhood of the point that maximizes tyre force (in order to minimize the vehicle's braking distance). In most situations, both objectives can be reached simultaneously if we are able to regulate wheel slip in the neighborhood of an appropriate setpoint.

In this paper, a new cascaded wheel-slip control strategy based on wheel slip and wheel acceleration measurements is presented. This new algorithm is able to stabilize globally and asymptotically the wheel slip around any prescribed setpoint, both in the stable and unstable regions of the tyre [6]. We show that this theoretical algorithm works in practice, by implementing it on the tyre-in-the-loop test facility of Delft University of Technology.

In the literature, one can mainly find two different kinds of anti-lock brake system designs: those based on logic switching from wheel deceleration information (see e.g. [3], [5], and [8]) and those based on wheel slip regulation (see e.g. [2], [7], [10]). There is, however, a third kind of algorithms that use both wheel slip and wheel acceleration measurements [9].

Anti-lock brake strategies based only on wheel deceleration information have quite interesting properties. Indeed, these strategies are very robust with respect to friction coefficient changes and can keep the wheel slip in a neighborhood of the optimal point, without using explicitly the value of the optimal setpoint. But a particularly unpleasant characteristic of these approaches is that they are often based on heuristic arguments, and thus tuning the thresholds involved in this kind of algorithms might be a difficult task [3]. Even if some recent results [5] give a first step towards a mathematical background for algorithms based on wheel deceleration thresholds (by an analysis of the stability of their limit cycles), these algorithms can only be used in order to track the optimal value of wheel slip λ_0. Namely, the value for which $\mu'(\lambda_0) = 0$. In other words, they are not able to stabilize the system around an arbitrary reference λ^* that belongs to the stable ($\lambda^* < \lambda_0$) or unstable region ($\lambda^* > \lambda_0$) of the tyre.

Approches based on pure wheel slip regulation have also quite interesting properties: the torque applied to the wheel converges to a fixed value (there are no periodic oscillations, like in wheel deceleration based algorithms [1]) and they work even if there is no clear maximum in the tyre characteristic. Their usage is nevertheless confronted to two difficulties. Firstly, they are mainly based on linearization arguments. The nonlinear system is linearized around the desired equilibrium point, and the stability analysis is thus only valid locally (see e.g. [7] and [9]). Secondly, they might fail to work in the unstable region of the tyre (see e.g. [10], where the control strategy generates a limit cycle if the setpoint is in the unstable domain). Thirdly, the available approaches are mainly based on pure feedback (there are no feedforward terms), which considerably limits the bandwidth of the closed-loop system.

Compared to the existing approaches, the main interest of our control algorithm is that it provides global asymptotic stability to the desired setpoint (independently of its location in the stable or unstable domain), by taking into account the non-linear nature of the system. Moreover, it gives precise bounds on the gains of the control law for which stability is proved mathematically [6]. Another original point of our approach is the feedforward term, which allows us to improve the bandwidth of the regulation scheme.

2. WHEEL DYNAMICS

The angular velocity ω of a given wheel of the vehicle has the following dynamics:

$$I\frac{d\omega}{dt} = -RF_x + T,$$

where I denotes the inertia of the wheel, R its radius, F_x the longitudinal tyre force, and T the torque applied to the wheel.

The longitudinal tyre force F_x is modelled by a relation

$$F_x(\lambda, F_z) = \mu(\lambda)F_z.$$

That is, by a function that depends linearly on the vertical load F_z and nonlinearly on the variable

$$\lambda = \frac{R\omega - v_x}{v_x},$$

which is called wheel slip. In this expression v_x denotes the longitudinal speed of the vehicle. It should be noted that this definition of slip shows a singularity at zero vehicle speed.

The nondimensional tyre characteristic $\mu(\cdot)$ is a skew-symmetric bounded curve, such that

$$\mu(0) = 0 \quad \text{and} \quad \mu'(0) > 0.$$

One of the simplest models for such a curve is given by

$$\mu(\lambda) = D\sin\left(C\arctan\left(B\lambda\right)\right),$$

which is a simplified form of Pacejka's magic formula [4]. The three coefficients B, C, and D are positive.

The tyre load F_z will be assumed to be constant and the vehicle will be supposed to brake with a deceleration $a_x(t)$. That is

$$\frac{dv_x}{dt} = a_x(t).$$

In simulations with a quarter-car vehicle model, we will take $a_x(t) = \mu(\lambda)g$, where g denotes the gravity.

If we define the variables x_1 and x_2 by

$$\begin{aligned} x_1 &= \lambda \\ x_2 &= R\frac{d\omega}{dt} - a_x(t), \end{aligned}$$

then we have the following dynamics

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{1}{v_x(t)}\left(-a_x(t)x_1 + x_2\right) \\ \frac{dx_2}{dt} &= -\frac{a\mu'(x_1)}{v_x(t)}\left(-a_x(t)x_1 + x_2\right) + \frac{u}{v_x(t)} - \frac{da_x}{dt}, \end{aligned}$$

where

$$a = \frac{R^2}{I}F_z \quad \text{and} \quad u = v_x(t)\frac{R}{I}\frac{dT}{dt}. \qquad (1)$$

Observe that we consider as a control variable the derivative of the torque applied to the wheel ; not the torque itself. Depending on the kind of technology used by the brake actuator (EMB or EHB), it might be necessary to integrate the control in order to have a brake torque reference.

3. CONTROL DESIGN

For a given wheel-slip reference $\lambda^*(t)$, we will define a filtered setpoint

$$\begin{aligned} \frac{d\lambda_1}{dt} &= \frac{\lambda_2}{v_x(t)} \\ \frac{d\lambda_2}{dt} &= \frac{-\gamma_1(\lambda_1 - \lambda^*) - \gamma_2\lambda_2}{v_x(t)}, \end{aligned}$$

where γ_1 and γ_2 are two positive real numbers.

The aim of this setpoint filter is twofold. On the one hand, it allows to have a smooth setpoint (that one can differentiate twice) event if the original setpoint is discontinuous (for exemple, piecewise constant). On the other hand, it allows to have a system for which all equations are divided by the vehicle's velocity. This homogeneity will allow us to analyse the system in a new (nonlinear) time-scale in which the dependence on speed disappears.

For further development we assume that a_x is constant and we apply a change of time-scale as in [5]. Let

$$s(t) = \int_0^t \frac{d\tau}{v_x(\tau)}.$$

This implies $dt = v_x(t)ds$ and, consequently, for any state-space trajectory $\varphi : \mathbb{R} \to \mathbb{R}^n$, we have

$$\frac{d\varphi}{ds} = \frac{d\varphi}{dt}\frac{dt}{ds} = \frac{d\varphi}{dt}v_x.$$

Therefore, defining $\dot{\varphi}(s) = \frac{d\varphi(s)}{ds}$ we obtain

$$\begin{aligned} \dot{x}_1 &= -a_x x_1 + x_2 && \text{(2a)} \\ \dot{x}_2 &= -a\mu'(x_1)(-a_x x_1 + x_2) + u && \text{(2b)} \\ \dot{\lambda}_1 &= \lambda_2 && \text{(2c)} \\ \dot{\lambda}_2 &= -\gamma_1(\lambda_1 - \lambda^*) - \gamma_2\lambda_2. && \text{(2d)} \end{aligned}$$

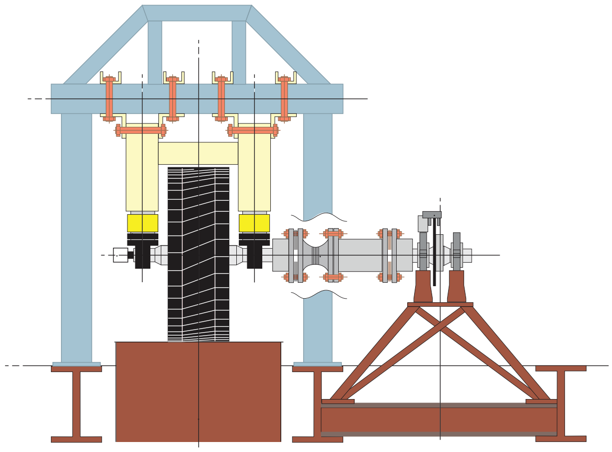

Fig. 1: Illustration of the tyre-in-the-loop setup.

Let $x_1^* = \lambda_1$ be the desired operating point for x_1, and define the error coordinates

$$z_1 = x_1 - x_1^* \tag{3a}$$
$$z_2 = x_2 - x_2^*. \tag{3b}$$

Now, let x_2 be a virtual control input in Equation (2a). Take $\alpha > 0$. If $x_2 = \lambda_2 + a_x x_1 - \alpha z_1$ then the closed-loop equation for z_1 reads

$$\dot{z}_1 = -\alpha z_1 + z_2, \tag{4}$$

which is exponentially stable if $z_2 = 0$. Hence, for x_2, we define the desired operating point as

$$x_2^* = \lambda_2 + a_x x_1 - \alpha z_1 \tag{5}$$

and design the control u so that x_2 converges towards x_2^* asymptotically.

This convergence can be obtained (see [6]) using the following control law

$$u = \lambda_3 + (a_x + a\mu'(x_1))\,\lambda_2 - k_1 z_1 - k_2 z_2, \tag{6}$$

where

$$\lambda_3 = -\gamma_1(\lambda_1 - \lambda^*) - \gamma_2\lambda_2.$$

Observe that while x_1^* is only based on $\lambda_1(t)$, the setpoint x_2^* is dynamic. The steady state part is $a_x x_1$, while the two other terms are used to decrease the error on z_1, both using cascaded feedback $-\alpha z_1$ and feedforward λ_2. Thanks to this dynamic setpoint, the system will converge to exactly the desired wheel slip, irrespectively from the tyre characteristic (unlike the existing algorithms [9]). In fact, one can prove [6] the following result.

Proposition 1 *Assume that there is a number c_μ^M such that*

$$|\mu'(s)| \leq c_\mu^M, \quad \forall s \in \mathbb{R}.$$

Then, when λ^* is constant, the origin $(z_1, z_2) = (0, 0)$ of the closed-loop system, defined by equations (2) and (6), is globally exponentially stable for sufficiently large values of k_1 and k_2.

A fundamental property of the previous result is that it works even if the estimated tyre characteristic $\hat{\mu}(\cdot)$ used by the control law differs from the actual tyre characteristic $\mu(\cdot)$. Indeed, this term is multiplied in the control law by λ_2, which vanishes asymptotically when λ^* is constant.

The constraint on the gains k_1 and k_2 is

$$k_2 \geq 2\left[\frac{k_1^2}{\alpha^2} - \frac{2k_1}{\alpha} + \eta_M^2 + 1\right] + \eta_m\,, \tag{7}$$

where η_m and η_M are such that $\eta_M \geq \eta(x_1) \geq -\eta_m$, the function η being defined by

$$\eta(x_1) = a\mu'(x_1) + a_x - \alpha.$$

We refer to [6] for the details of the proof.

4. SIMULATION RESULTS

The first aim of our simulations is to observe the effects of the feedback and feedforward terms. In accordance with both intuition and theoretical study, the following phenomena can be observed:

1. When the feedback gains are equal to zero (that is, in the case of pure feedforward control), the system tracks the desired wheel slip reference only if this reference is in the tyre's stable zone; otherwise the purely open-loop system is unstable (Figure 4).
2. When the feedback gains satisfy condition (7) and the feedforward is not used (that is, in the case of pure feedback control), the system tracks the desired wheel slip reference, but with a very poor performance during transients (Figure 5).
3. When both feedback and feedforward terms are included, the system follows exactly the filtered reference (Figure 6). But this, only if there are no perturbations on the system (like delays or uncertainties on the system's parameters).

The second aim of our simulations is to observe the effects of perturbations, in order to evaluate the robustness of our control laws (when both feedback and feedforward terms are used). We considered mainly three cases:

1. When a pure delay is introduced in the control loop (take, for example, the case of a typical hydraulic actuator delay of 15 ms) the performance remains good and the system remains stable, provided that the delay is not too big (Figure 7).
2. When the system's parameters used in the control law do not match those of the true system (like, for exemple, a change of tyre characteristics) the system remains stable, but the performance is considerably reduced (Figure 8). But an adaptation law can be developed to improve this weakness [6].

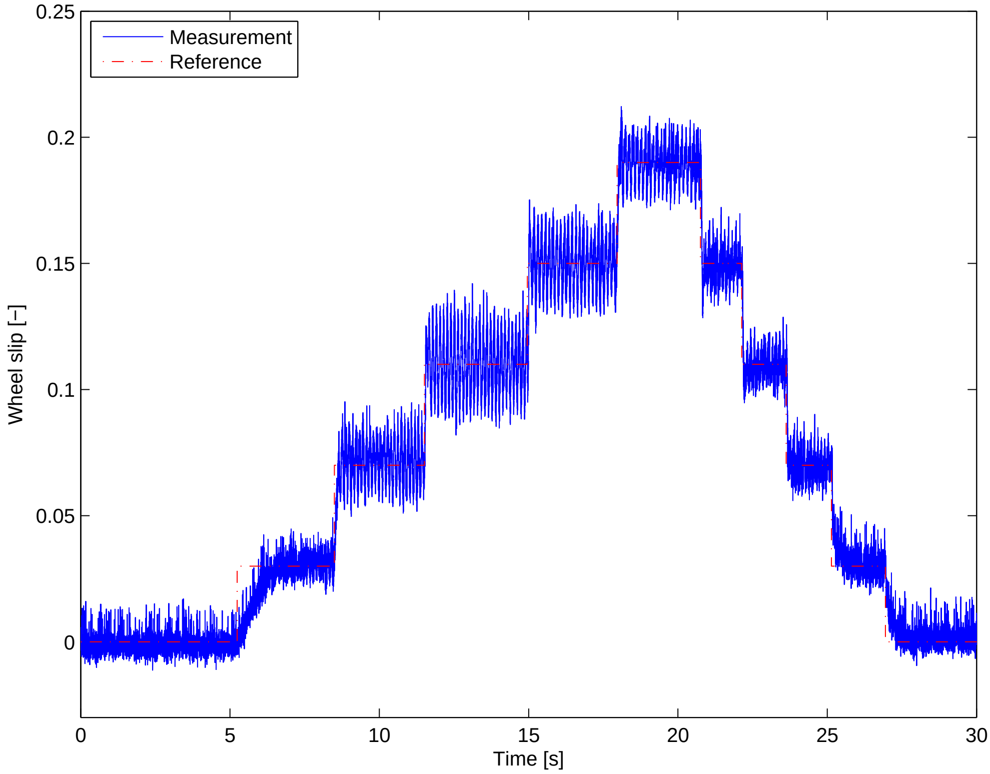

Fig. 2: Experimental validation of our our controller, when only the feedback is enabled (no feedforward).

3. When both pure delay and parameter uncertainties are considered, the results are quite close to the case of pure parameter uncertainties (Figure 9).

In order to prepare the experimental validation, the controller has also been simulated on the more complex model developed in [1]. The results of the simulations are perfectly comparable to the experimental results of the next section, and are therefore skipped here.

5. EXPERIMENTAL RESULTS

The tyre-in-the-loop experimental facility of Delft University of Technology on which the ABS is tested consist of a large steel drum of 2.5 meter diameter on top of which the tyre is rolling (see Figure 1). The setup has been used for many years for tyre modeling and identification using open-loop excitation (see [4], [11], and the references therein). Recently, the electronics was upgraded in order to allow closed-loop tests to be performed and, in particular, rapid prototyping and testing of ABS controllers. The drum is driven by a large electromotor and can run up to 300 km/h. The speed of the drum can be accurately measured thanks to an encoder. The weight of the drum makes it more suitable for keeping a constant speed.

The wheel (together with the tyre) is attached to an axle with a rigidly constrained height. The axle is supported by two bearings on both side of the wheel. The bearing housings are connected to the fixed frame by means of piezo-electric force transducers. An hydraulic disk brake is mounted on one side of the axle. The pressure in the calliper is locally controlled by an analog electronics module, which is connected to a servo-valve in order to match the reference pressure given by the dSpace computer. An encoder with 5000 teeth is mounted on the other side of the axle to provide very accurate wheel speed measurements.

Two experiments have been performed on the setup. During the first one, the slip reference is increased or decreased by steps of 4% from 0 up to 20% which is already in the unstable region of the tyre. Only the feedback term si enabled, not the feedforward one. The results are presented on Figure 2. It can be observed that the controller drives the slip precisely towards the reference value, both in the stable and unstable region of the tyre. It is interesting to note that the oscillations in the slip are larger when the slip is higher. This is linked to the fact that the damping provided by the tyre is decreasing when the brake force is approaching the saturation. The same phenomena leads to a decrease of the relaxation length at high slip, as observed in [11].

During the second test, performances with and without feedforward are compared. During the first 10 seconds, only the feedback is enabled. Then the feedforward is turned on. On Figure 3, it can be observed that the convergence to a new reference is much faster with the feedforward on. This is particularly noticeable at low slip, when the controller with feedback only is particularly slow.

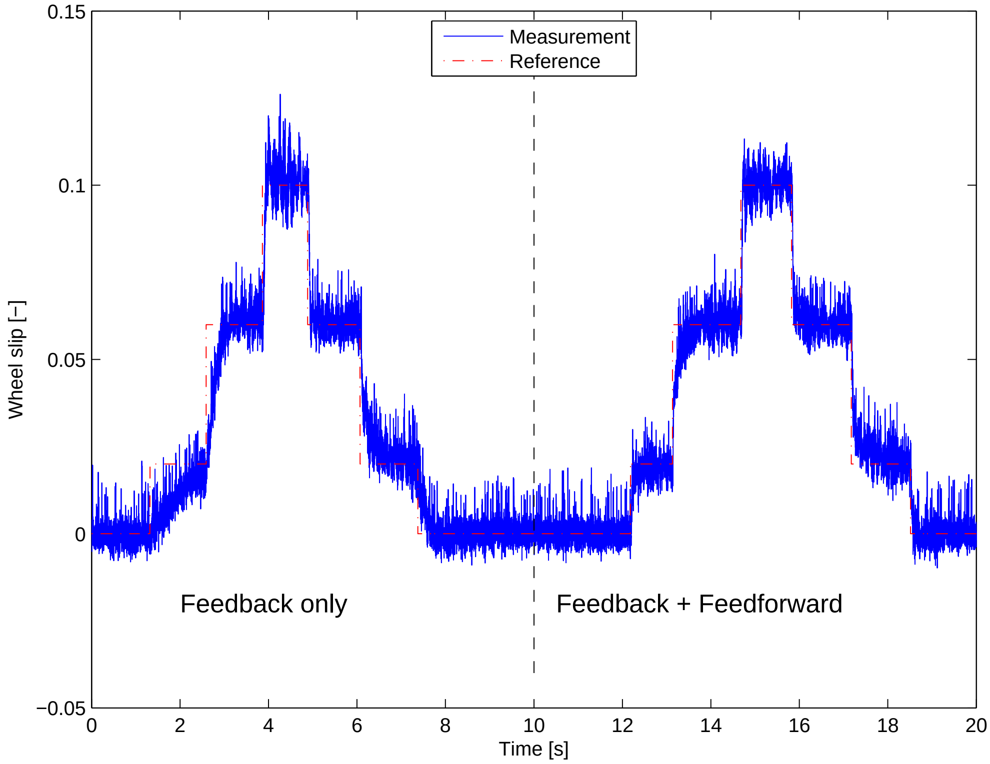

Fig. 3: Comparison of the performances with and without feedforward.

6. CONCLUSION

In this paper, a new cascaded wheel slip control strategy based on wheel slip and wheel acceleration measurements was presented. The theory developed in [6] predicted that the control law could stabilize globally and asymptotically the wheel slip around any prescribed setpoint, both in the stable and unstable regions of the tyre, based on a simple model. This paper supports the theory by showing a practical implementation and validation of the controller on a tyre-in-the-loop test facility. Prior to implementation, simulations where used to assess robustness to time delays (in particular to a typical 15ms hydraulic actuator delay) and to uncertainties in the tyre-road friction. The first experimental test shows the convergence of the slip towards the reference, also in the unstable zone. The second test concludes that the feedforward is significantly increasing the speed of convergence, in particular in the stable zone.

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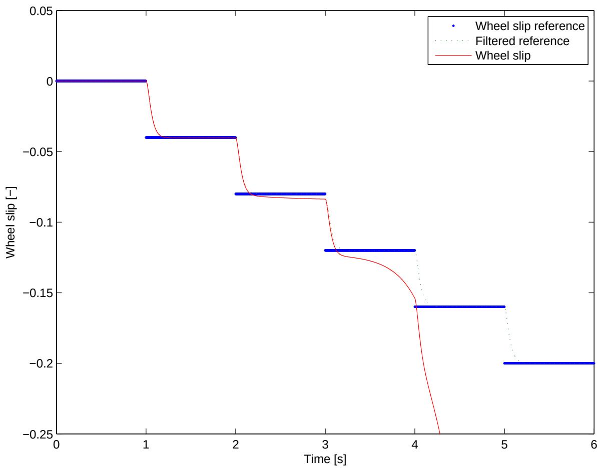

Fig. 4: Pure feedforward control ($k_1 = 0$ and $k_2 = 0$).

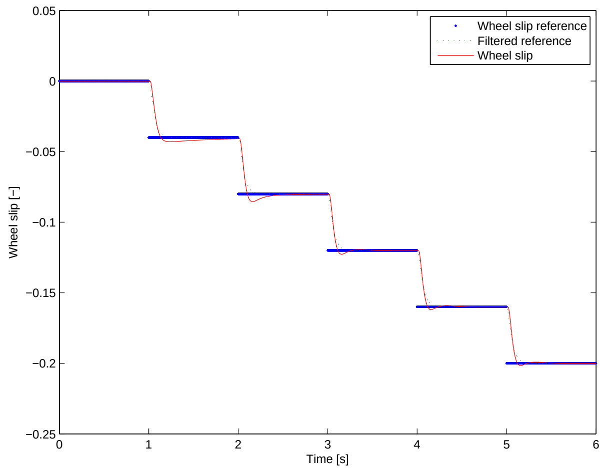

Fig. 7: Combined control, with a delay of 15 ms.

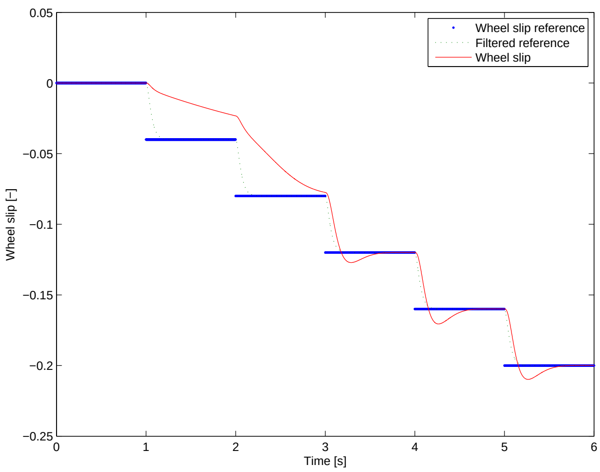

Fig. 5: Pure feedback control.

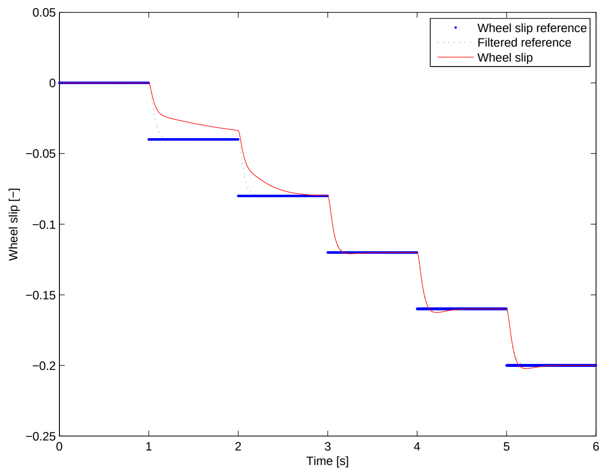

Fig. 8: Combined control with a perturbation of $\mu(\cdot)$.

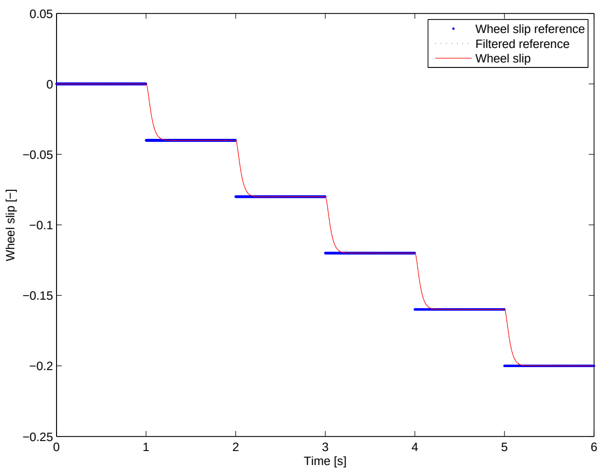

Fig. 6: Combined feedback and feedforward control (without perturbations).

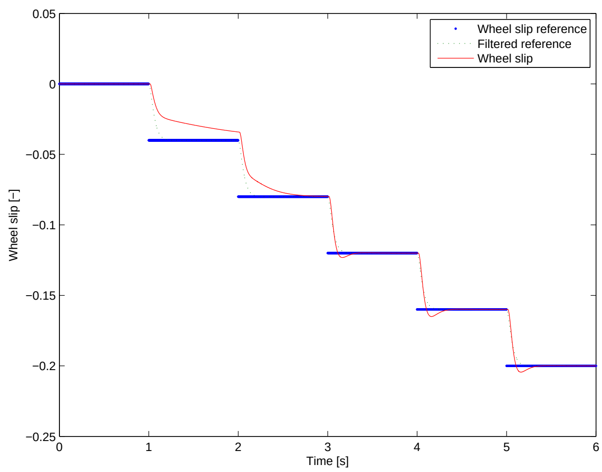

Fig. 9: Combined control, with a delay of 15 ms and a perturbation of $\mu(\cdot)$.